

# Fiera Capital's CIA Accounting Discount Rate Curve – Implementation Note

## Fiera Capital Corporation

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# 1 Background

In September 2011, the Canadian Institute of Actuaries (“CIA”) retained the services of Fiera Capital Corporation (“Fiera Capital”) to produce, on a monthly basis, the Fiera Capital’s CIA Accounting Discount Rate Curve that can be used by sponsors to select the appropriate accounting discount rate to value pension and other post-employment benefits plans liabilities. In its original version, this yield curve was derived from the approach suggested by the CIA and described in the official document ‘*Educational Note, Accounting Discount Rate Assumption for Pension and Post-Employment Benefit Plans*’ (“Educational Note”), published in September 2011.

At that time, three approaches were considered by the CIA for the accounting discount rate curve construction and Approach C from the Educational Note was retained:

“For purposes of developing the yield curve, AA-rated corporate bonds are used for maturities up to 10 years since the market is sufficiently deep at these maturities. For maturities greater than 10 years, the yield curve is extrapolated using AA-rated Canadian provincial bonds. In order to reflect the difference in credit risk between AA-rated corporate bonds and AA-rated provincial bonds, a spread adjustment is added to the provincial bond yields.”

As part of the curve construction process, an approach was developed by the CIA to calculate the spread adjustment to be added to the provincial bonds yields with maturities greater than 10 years. That approach, which is described in detail in the 2011 Educational Note as well as in [Fiera Capital’s Implementation Note from 2011](#), heavily relied on information from AA-rated corporate bonds with maturities greater than 10 years. Following changes in the Canadian bond market environment over the last few years, particularly with regards to the significant reduction in the number of AA-rated corporate bonds with maturities greater than 10 years, the CIA decided to revisit the yield curve construction process in order to further improve the extrapolation approach and the spread adjustment calculation for bonds with maturities greater than 10 years.

This document describes the revised approach developed by the CIA, after considering a number of possible approaches, and used by Fiera Capital in developing the Fiera Capital’s CIA Accounting Discount Rate Curve starting from November 2016. Information on the previous approach used to prepare the Fiera Capital’s CIA Accounting Discount Rate Curve before November 2016 can be found in the [Fiera Capital’s Implementation Note from 2011](#).

For the purpose of calculating pension and other post-employment benefits plans liabilities, yields used to discount a projected stream of pension benefit payments should be based on zero coupon bond yields. Hence, the term ‘spot curve’ is used in reference to Fiera Capital’s CIA Accounting Discount Rate Curve.

## 2 Securities Selection

The first step in the spot curve construction is securities selection. For this purpose, three subsets are defined: a Canada Bond subset, a Provincial Bond subset and a Corporate Bond subset. The Bank of America Merrill Lynch Canada Broad Bond Market Index<sup>1</sup>, which is considered to be representative of the Canadian bond market, is used as the starting point to determine these three subsets. The pricing methodology used in Bank of America Merrill Lynch Canada Broad Bond Market Index is considered to appropriately reflect market valuation. The Bank of America Merrill Lynch Canada Broad Bond Market Index is compiled using bond valuations provided by Statpro.

In light of the 2011 Educational Note's recommendation to include several representative criteria of the Canadian fixed income market, a filter is applied to the Bank of America Merrill Lynch Canada Broad Bond Market Index to remove all bonds:

- with non-standard cash flow structures such as “amortizers” and/or “sinking”;
- with explicit callable options (the Canada call provision as an explicitly callable bond is not considered for the purpose of this filter); and
- which have less than \$100,000,000 of outstanding nominal amount in the index.

In addition, in order to be included in the subsets, the following criteria must be met:

- for the Canada Bond subset, issuers must not be Canadian federal agencies;
- for the Provincial Bond subset, issuers must have a credit rating greater or equal to AA by at least one major rating agency<sup>2</sup> and must be direct provincial issuers (e.g. Ontario, British-Columbia, Alberta, etc.);
- for the Corporate Bond subset, issuers must have a credit rating equal to AA by at least one major rating agency<sup>2</sup> and must not be quasi-governmental entities (e.g. Port and Airport Authorities, Hospitals, Universities, etc.).

The filtering characteristics of each issue in the index are drawn from Bank of America Merrill Lynch Canadian Broad Market Index characteristics. It is important to note that the filters may change over time to reflect market conditions. Any change to these filters will be discussed and agreed upon by the CIA and Fiera Capital.

## 3 Yield to Maturity Observations for the Spot Curve Extraction

As described in the 2011 Educational Note under Approach C, the yield to maturity observations for the spot curve extraction are derived from two groups of bonds:

1. The yield to maturity  $Y_i^C$  for each corporate bond  $i$  from the Corporate Bond subset, where  $Y_i^C$  is the quoted yield to maturity of the bond (mid-market) expressed as a semi-annually compounded rate; and

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<sup>2</sup> The credit ratings agencies used to determine the credit rating quality criteria are: Standard & Poor's (S&P), Moody's, Fitch Group and Dominion Bond Rating Service (DBRS).

- The adjusted yield to maturity  ${}^*Y_i^P$  for each provincial bond  $i$  from the Provincial Bond subset with a maturity greater than 10.5 years at construction date, defined as

$${}^*Y_i^P = Y_i^P + P_i,$$

where  $Y_i^P$  is the quoted yield to maturity of the bond (mid-market) expressed as a semi-annually compounded rate and  $P_i$  is the provincial spread adjustment of the provincial bond  $i$ , as described in section 4.

## 4 Calculation of the Provincial Spread Adjustments

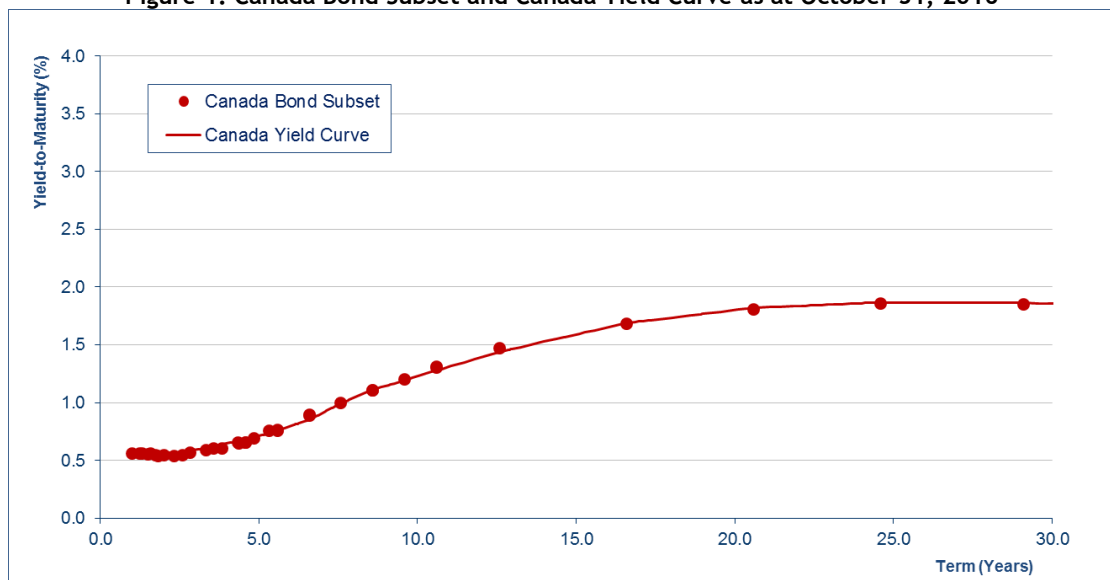
This section describes the approach used to derive the provincial spread adjustments  $P_i$  to add to the observed yield to maturity of provincial bonds with maturities greater than 10.5 years to account for credit risk of AA-rated corporate bonds.

### 4.1 Preparation of the Generic Canada Yield Curve

The first step in the calculation of the provincial spread adjustments is to derive a generic Canada yield curve, which will subsequently be used to calculate the spreads of individual provincial and corporate bonds over this curve.

The methodology Fiera Capital uses to derive this generic Canada yield curve is based on a kernel smoothing method. This results in a smoothed yield curve as a function of term to maturity using the Canada Bond subset, as shown in Figure 1. The smoothing method ensures that the yield curve is approximately defined at each term to maturity.

Figure 1: Canada Bond Subset and Canada Yield Curve as at October 31, 2016



## 4.2 Calculation of the Average Corporate Spread

In order to compute the average corporate spread over the Canada yield curve, bonds with maturities between 4.5 and 10.5 years from the Corporate Bond subset are used.

For each corporate bond  $i$  with a maturity between 4.5 and 10.5 years, its spread  $S_i^C$  is computed relative to the generic Canada yield curve described in section 4.1 as

$$S_i^C = Y_i^C - \varphi(T_i^C),$$

where  $Y_i^C$  is the quoted yield to maturity of the bond (mid-market) expressed as a semi-annually compounded rate and  $\varphi(T_i^C)$  is the semi-annually compounded yield to maturity at maturity of the corporate bond  $T_i^C$  obtained from the generic Canada yield curve methodology described in section 4.1.

The average corporate spread over the Canada yield curve  $\bar{S}^C$  is computed as

$$\bar{S}^C = \frac{1}{N^C} \sum_{i=1}^{N^C} S_i^C,$$

where  $N^C$  is the number of corporate bonds with maturities between 4.5 and 10.5 years.

## 4.3 Calculation of the Average Provincial Spread

In order to compute the average provincial spread over the Canada yield curve, bonds with maturities between 4.5 and 10.5 years from the Provincial Bond subset are used.

For each provincial bond  $i$  with a maturity between 4.5 and 10.5 years, its spread  $S_i^P$  is computed relative to the generic Canada yield curve described in section 4.1 as

$$S_i^P = Y_i^P - \varphi(T_i^P),$$

where  $Y_i^P$  is the quoted yield to maturity of the bond (mid-market) expressed as a semi-annually compounded rate and  $\varphi(T_i^P)$  is the semi-annually compounded yield to maturity at maturity of the provincial bond  $T_i^P$  obtained from the generic Canada yield curve methodology described in section 4.1.

The average provincial spread over the Canada yield curve  $\bar{S}^P$  is computed as

$$\bar{S}^P = \frac{1}{N^P} \sum_{i=1}^{N^P} S_i^P,$$

where  $N^P$  is the number of provincial bonds with maturities between 4.5 and 10.5 years.

## 4.4 Calculation of the Spread Ratio $R$

The spread ratio  $R$  is defined as the ratio of the average corporate spread to the average provincial spread, as defined respectively in sections 4.2 and 4.3 and as shown below:

$$R = \frac{\bar{S}^C}{\bar{S}^P}$$

## 4.5 Calculation of the Provincial Spread Adjustments $P_i$

For each provincial bond  $i$  from the Provincial Bond subset with a maturity greater than 10.5 years at construction date, the provincial spread adjustment  $P_i$  is calculated as:

$$P_i = [Y_i^P - \varphi(T_i^P)]^*(R-1),$$

where  $Y_i^P$  is the quoted yield to maturity of the bond (mid-market) expressed as a semi-annually compounded rate,  $\varphi(T_i^P)$  is the semi-annually compounded yield to maturity at maturity of the provincial bond  $T_i^P$  obtained from the generic Canada yield curve methodology described in section 4.1, and  $R$  is the spread ratio defined above.

## 5 Fiera Capital Spot Curve Extraction Methodology

The objective of the spot curve extraction methodology is to derive the most representative spot curve from CIA's yield to maturity adjusted observation set, as defined in section 3. Fiera Capital's analysis tested various parametric and non-parametric models to achieve the best balance between the different criteria described below. As a result, a parametric approach for deriving the spot curve was adopted.

The following criteria will govern the curve extraction methodology:

- The spot curve should avoid having negative forward rates;
- The spot curve should produce a good fit to the CIA's yield to maturity adjusted observations. The mathematical criteria used in Fiera Capital's methodology is to minimize the overall pricing error (i.e. the difference between the adjusted observed price and its price obtained through the discounting of its future cash flows with the spot curve);
- The chosen methodology should exhibit enough flexibility to enable various curve shapes;
- The spot curve should be smooth (e.g. mathematically the second order finite difference approximation);
- The model implied short term rate and ultimate long term rate must be reasonable; and
- The spot curve model must be stable over time.

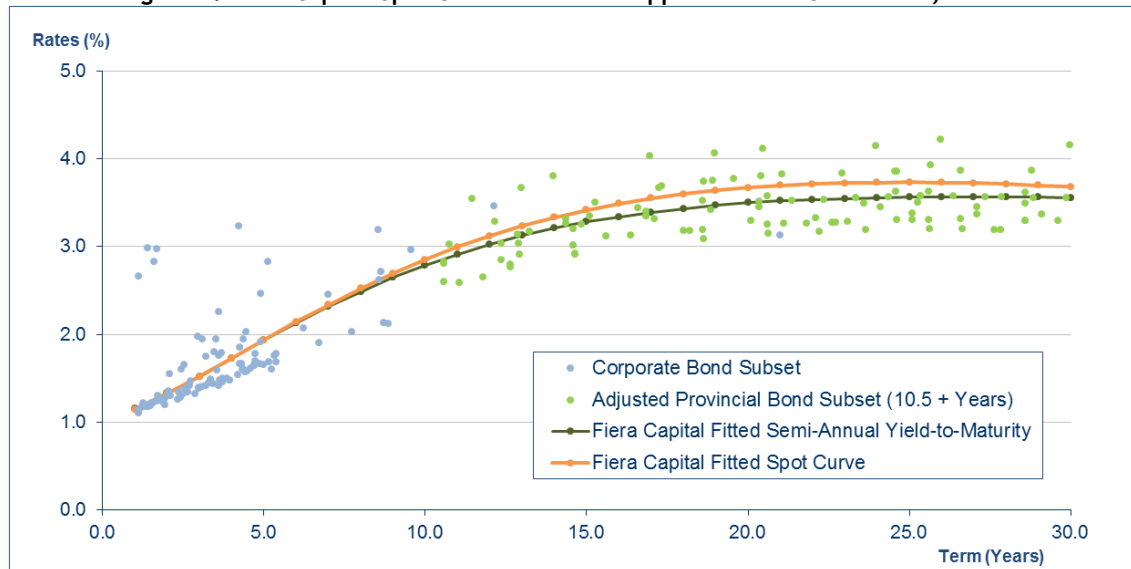
Fiera Capital's spot curve extraction methodology can qualitatively be described by the following optimization program:

- An optimal parametric specification by minimizing the sum of squared pricing differences (i.e., the norm) between the adjusted observed prices and the theoretical prices obtained by discounting the future cash flows with the adopted parametric spot curve model;
- The price difference norm uses a different weighting scheme than the usual "1/N" weights in order to obtain a better fit to the adjusted observed market prices that is based on the duration of each bond; and
- The parametric specification is constrained through constraints on the short and long term spot rates obtained by the adopted parametric spot curve model.

The quantitative procedure required to fit the adopted parametric yield curve model is based on non-linear least squares regression techniques involving non-linear optimization routines. All the quantitative models and tools involved in the spot curve extraction methodology are developed by Fiera Capital.

The following figure shows, as at October 31, 2016, the yields to maturity  $Y_i^C$  from the Corporate Bond subset and the adjusted yields to maturity  $Y_i^P$  from the Provincial Bond subset with maturities greater than 10.5 years, as defined in section 3. The figure also shows the fitted semi-annual yield-to-maturity curve and the fitted spot curve as at October 31, 2016 based on the approach described above.

Figure 2: Fiera Capital Spot Curve Extraction Approach as at October 31, 2016



## 6 References

- Canadian Institute of Actuaries (2011), *Accounting Discount Rate Assumption for Pension and Post-Employment Plans*, Educational Note by the Task Force on Pension and Post-retirement Benefit Accounting Discount Rates, <https://www.cia-ica.ca/docs/default-source/2011/211088e.pdf>.
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- Canadian Institute of Actuaries (2018), *Setting the Accounting Discount Rate Assumption for Pension and Post-Employment Plans*, Revised Educational Note by the Committee on Pension and Post-retirement Benefit Accounting Discount Rates, <https://www.cia-ica.ca/docs/default-source/2018/218086e.pdf>.